Patterns, Symmetry, and Beauty: Teacher's Guide

Grade Level: 7-12	**Curriculum Focus:** Mathematics	**Lesson Duration:** Two class periods

Program Description

What is beauty? Why do certain proportions and patterns evoke a pleasant feeling? Mathematicians and artists alike agree that beauty can be found in nature and in art, architecture, and music. From the rhythms of Wynton Marsalis to the patterns of the Fibonacci sequence found in shells, beauty has a certain mathematical truth to it.

Onscreen Questions

Before watching the video

- It's no coincidence that people find familiar shapes and patterns appealing. Our minds easily recall simple geometric forms, such as the circular shape of the moon or the triangle that frames a tree. As you watch, note examples of how simple mathematical relationships play a role in what appears beautiful.

After watching the video

- Look for examples of the Fibonacci sequence and the golden section in nature.
- What are some examples of the Fibonacci sequence and the golden section in your immediate environment? How are these relationships expressed?

Lesson Plan

Student Objective

- Understand the Fibonacci sequence (numerically, algebraically, and geometrically).
- Understand how the Fibonacci sequence is expressed in nature.

Materials

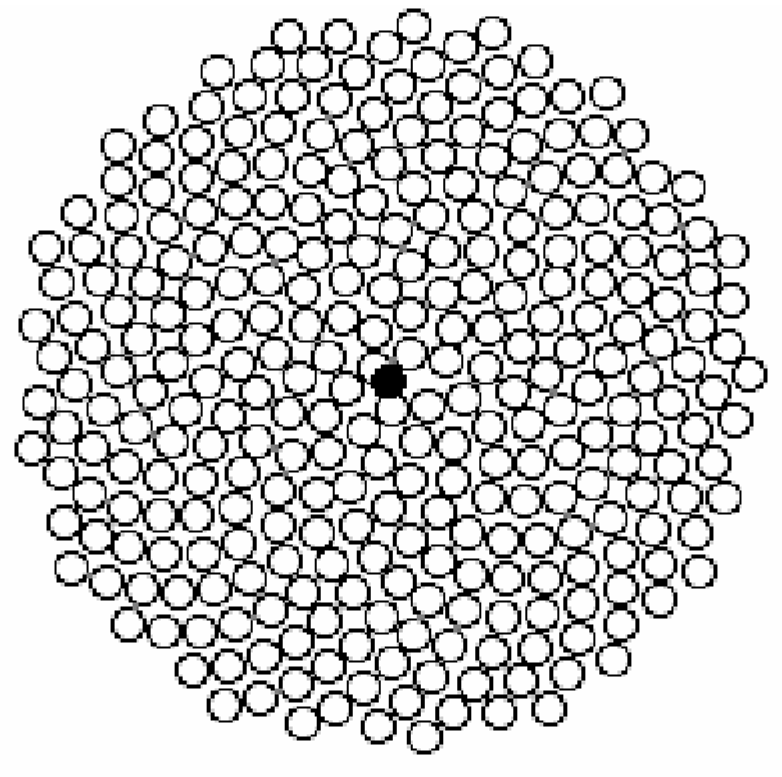
- Copies of the Creating the Fibonacci Spiral handout
- Computers with Internet access (optional but very helpful)

Procedures

1. Begin by discussing the Fibonacci sequence, which was first observed by the Italian mathematician Leonardo Fibonacci in 1202. He was investigating how fast rabbits could breed under ideal circumstances. He made the following assumptions:
   - Begin with one male and one female rabbit. Rabbits can mate at the age of one month, so by the end of the second month, each female can produce another pair of rabbits.
   - The rabbits never die.
   - The female produces one male and one female every month.
2. Work with the class to see if students can develop the sequence themselves. Remind them that they're counting pairs of rabbits (the number in parentheses), not individuals. Walk them through the first few months of the problem:
   (1.) Begin with one pair of rabbits. (1)
   (2.) At the end of the first month, still only one pair exists. (1)
   (3.) At the end of the second month, the female has produced a second pair, so two pairs exist. (2)
   (4.) At the end of the third month, the original female has produced another pair, and now three pairs exist. (3)
   (5.) At the end of the fourth month, the original female has produced yet another pair, and the female born two months earlier has produced her first pair, making a total of five pairs. (5)
3. Write the pattern that has emerged in step 2 on the board:

   1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233...

   Discuss the sequence: Help students understand that to get the next number in the sequence, you add the previous two numbers. This is the Fibonacci sequence. The term that mathematicians use for the type of rule followed to obtain the numbers in the sequence is algorithm. As a class, continue the sequence for the next few numbers.
4. Tell students that the Fibonacci sequence has intrigued mathematicians for centuries. What's more, mathematicians have noticed that these numbers appear in many patterns in nature, often creating the beauty. Tell students that they are going to look for Fibonacci numbers in objects from nature. Make sure that students understand that they are looking for specific numbers that appear in the sequence, not for the entire sequence.
5. Divide students into groups of three or four. Show the following diagram of a seed head from a sunflower plant. Tell students to look for Fibonacci numbers; make sure a list of the numbers is on the board. For more information, go to http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html



6. Each small circle on the illustration represents a seed, all of which form a spiral pattern from the center. Use two different colored pencils to mark the clockwise and counterclockwise spirals. Starting with an outside circle, trace the spiral shape through the circles that define one spiral. (Note: Some circles will not be included.)
7. Count the clockwise spirals, then count the counterclockwise spirals. The numbers should be consecutive Fibonacci numbers. (For this figure the Fibonacci numbers are 21 and 34.) Explain to students that seed heads in nature arrange themselves in Fibonacci spirals.
8. Assign the classroom activity (see last page) and have students share their drawings. Then discuss how rectangles with Fibonacci dimensions are used in art and architecture. You may use an example from artist Piet Mondrian, who used three- and five-unit squares in his paintings. Also, the ancient Egyptians used Fibonacci dimensions in the Great Pyramid at Giza, and the Greeks used them in the Parthenon. Brainstorm with students to name animals that have spiral shapes, such as snail and nautilus shells. (Some scientists think the spiral protects an animal inside its shell.)

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Active participation in classroom discussions; ability to work cooperatively to complete the in-class activity.

- **2 points:** Some degree of participation in classroom discussions; ability to work somewhat cooperatively to complete the in-class activity.
- **1 point:** Small amount of participation in classroom discussions; attempt to work cooperatively to complete the in-class activity.

Vocabulary

algorithm

Definition: A step-by-step procedure for solving a problem

Context: The algorithm for obtaining the numbers in the Fibonacci sequence is to add the previous two terms together to get the next term in the sequence.

logarithmic spiral

Definition: A shape that winds around a center and recedes from the center point with exponential growth

Context: The nautilus shell is an example of a logarithmic spiral.

sequence

Definition: A set of elements ordered in a certain way

Context: The terms of the Fibonacci sequence become progressively larger.

term

Definition: An element in a series or sequence

Context: The mathematician Jacques Binet discovered that he could obtain each of the terms in the Fibonacci sequence by inserting consecutive numbers into a formula.

Academic Standards

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit link: http://www.mcrel.org/compendium/browse.asp

This lesson plan addresses the following national standards:

- Mathematics— Understands and applies basic and advanced properties of the concepts of numbers, Understands and applies basic and advanced properties of the concepts of geometry, Understands the general nature and uses of mathematics

The National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to http://standards.nctm.org/.

This lesson plan addresses the following math standards:

- Algebra
- Geometry

Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- http://school.discovery.com/teachingtools/teachingtools.html

Classroom Activity

Creating the Fibonacci Spiral

Materials

- sheet of quarter-inch grid paper
- ruler
- compass

Follow the directions and watch the spiral emerge. (Each successive square will have one edge with a length the sum of the two squares immediately preceding it.)

(1.) Draw a 1-inch square. Draw a second 1-inch square to its left, making sure the squares touch.

(2.) Draw a 2-inch square above the two 1-inch squares, touching the lower squares.

(3.) Draw a 3-inch square to the right of the three existing squares; its left side should touch the other squares.

(4.) Draw a 5-inch square below these squares.

(5.) Draw an 8-inch square to the left of the existing five squares.

(6.) Draw a 13-inch square above the six squares.

(7.) Use a compass to complete the drawing. Within each square of your drawing, draw an arc from one corner to the opposite corner. (Each arc will have a radius equal to the length of one side of its square.) Connect each arc to the next. To begin, place your pencil in the upper-right corner of the original 1-inch square and draw an arc toward the lower-left corner. In the second square, draw an arc from that point (the lower-right corner) to the upper-left corner of the second square. Continue drawing arcs in each square, starting each arc at the point where the last one ended.

(8.) You will create a logarithmic spiral. What forms in nature reflect this shape?