DISCOVERING ALGEBRA WITH GRAPHING CALCULATORS
Understanding Exponential Functions
Teacher's Guide

Grade Level: 9–12 **Curriculum Focus:** Math **Running Time:** 25 minutes

Program Description

Investigates the properties of exponential functions and graphs. Students learn the algebra of exponential functions including growth and decay. A banker supplies "interesting" information on how money accrues in bank accounts.

Learning Objectives

After viewing the program and participating in discussion, students will be able to:

- Describe the graph of an exponential function based on the value of the exponent's base;
- Recognize the general form of an exponential function as $y = Ab^t$;
- Relate exponential functions to fractals and their design;
- Solve simple and compound interest problems using the graphing calculator;
- Use the irrational number *e* in exponential decay problems;
- Apply algebra to real-world situations and develop logical reasoning skills.

Classroom Connections

How is an exponent's base value related to the growth of the function? Would $y = 3^x$ or $y = 15^x$ illustrate a faster growth rate?

Why is the y-intercept always one in exponential functions? Why is 0^x undefined?

Why are the domain and range of exponential functions limited to positive, real numbers? Is it possible to have an exponential function that exists below the x-axis?

Research the Sierpinski Triangle to discover its unique properties:

The Magic Sierpinski Triangle

http://serendip.brynmawr.edu/playground/sierpinski.html

The Sierpinski Triangle Fractal
http://www.arcytech.org/java/fractals/sierpinski.shtml

Math Forum: Pascal's Triangle
http://mathforum.org/workshops/usi/pascal/pascal_sierpinski.html

How do fractals and the Sierpinski Triangle relate to exponential functions?

Classroom Activities

As a class, visit the Federal Student Aid Web site (http://www.ed.gov/offices/OSFAP/DirectLoan/calc.html) to find out the current interest rate for federally-subsidized Stafford student loans. Assume that a student needs to take out a $20,000 loan to pay for college. The interest is calculated yearly and the student has 11 years to pay off the loan. Determine the function and graph it on the calculator. What is the total amount of interest accrued on the loan after 11 years?

Exponential decay is often used to illustrate radioactive decay; radioactive substances tend to be very unstable, and the elements or compounds break down according to a specific formula. The equation for the model is $A = A_0e^{kt}$ (where k is a negative number representing the rate of decay, e is the base of the natural logarithm, and A_0 is the original amount present at time $t = 0$.) Tell students you have 50 grams of a radioactive compound. If the compound decays at 0.02 percent per year, what is the function that explains this relationship? How much of the original compound would be left after 100 years?

Have students pair up to work the following problem. The pairs will use the "TVM Solver" feature and a graph to find the solutions, although this problem requires an additional step beyond the sample problem covered in the program:

Two friends want to start saving up for private jets. Javier invests $200 a month in a mutual fund that averages 9 percent interest annually. After 10 years, he stops investing his $200 monthly contribution and allows the interest to accrue on his current balance for another seven years. Sam waits three years before he begins saving and then adds $300 a month to an account that earns 10 percent interest annually. After seven years, he stops adding money and allows the interest to accrue for another seven years.

-When Javier stops adding money to his account, how much has he saved?
-How much money has Javier earned after the additional seven years?
-How much money has Sam saved after seven years?
-How much money does Sam have after the additional seven years?
-Did you expect these results? What could explain the difference in interest earned?

Target Vocabulary*

coefficient - a constant factor of a term as distinguished from a variable

domain - the set of elements to which a mathematical or logical variable is limited; specifically: the set on which a function is defined

exponent - a symbol written above and to the right of a mathematical expression to indicate the operation of raising to a power

fractal - any of various extremely irregular curves or shapes for which any suitably chosen part is similar in shape to a given larger or smaller part when magnified or reduced to the same size

function - a: a mathematical correspondence that assigns exactly one element of one set to each element of the same or another set; b: a variable (as a quality, trait, or measurement) that depends on and varies with another

quadratic - involving terms of the second degree at most (*quadratic* function) (*quadratic* equations)

range - a: the set of values a function may take on; b: the class of admissible values of a variable

y-intercept - the y-coordinate of a point where a line, curve, or surface intersects the y-axis

*All definitions from Merriam-Webster Online: http://www.m-w.com

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Academic Standards

The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to http://standards.nctm.org/.

This lesson plan addresses the following math standards:

- Represent and analyze mathematical situations using algebraic symbols
- Understanding patterns, relations, and functions