

Discovering Math: Intermediate Probability

Teacher's Guide

Grade Level: 6–8

Curriculum Focus: Mathematics

Lesson Duration: 2–3 class periods

Program Description

Discovering Math: Intermediate Probability – From theoretical models to simulations and experiments to certainty measurements, introduce students to more advanced concepts of probability.

Lesson Plan

Student Objectives

- Create the sample space to show all possible outcomes of an event.
- Calculate the theoretical probability of specific events.
- Calculate the experimental probability of specific events.
- Given theoretical probability statements, create a spinner that will match the statements.

Materials

- *Discovering Math: Intermediate Probability* video
- Two bags of colored cubes (2 red cubes, 2 green cubes, and 1 blue cube in each bag) for each pair of students
- Theoretical and Experimental Probability Worksheet (see below)
- Spinner Template (see below)
- Probability Statements Worksheet (see below)
- Paper clips

Procedures

1. Display 10 colored cubes for the students (5 blue, 2 red, 3 green). Ask students the probability of choosing one of the cubes (e.g., What is the probability of choosing a green cube?).
2. Tell students they will be creating a sample space and calculating theoretical and experimental probabilities. Review theoretical and experimental probability.

- theoretical probability — When all outcomes are equally likely, the theoretical probability of an event is the ratio of the number of favorable outcomes to the number of possible outcomes.
 - experimental probability — A probability based on the results of repeated trials of an experiment. The experimental probability of an event is expressed as the number of favorable outcomes over the number of trials or items in a sample.
3. Assign each student a partner. Distribute two bags of colored cubes and a Probability Worksheet to each pair. Tell them what each bag contains.
- Have students create the sample space to show all possible outcomes for choosing one cube from each bag. They can use a tree diagram or a grid to create the sample space.
 - Using the sample space, have students list the probability of each possible outcome (e.g., red, red = $4/25$).
 - Ask them use the probabilities to calculate how many of each outcome might occur in 50 picks.
 - Have students conduct an experiment by choosing one cube from each bag, recording the outcome, and replacing the cubes. Repeat 50 times.
 - Ask them to use their data to calculate the experimental probability of each outcome after 50 picks.
 - Have students should compare the theoretical and experimental probabilities and make a conclusion statement (e.g., the theoretical probability was greater than the experimental probability).
4. Tell students they will create a spinner that will produce outcomes that should match a series of probability statements.
- Assign each student a partner. Distribute the Probability Statements Worksheet and Spinner Template to each pair.
 - Have students read and discuss the probability statements.
 - Allow time for each pair to create their spinner.
 - Have students spin their spinners and record the outcome. Repeat the procedure 50 times.
 - Ask students to use their data to calculate the experimental probability for each outcome after the 50 spins.
 - Have them compare the experimental probabilities to the probability statements to see if their spinner produced the correct results. If they find that their results do not match the probability statements, have them check their spinners (some students may need to create new spinners) and then conduct 50 additional spins.
 - Remind students that the greater the number of trials in an experiment, the closer the experimental probability will be to the theoretical probability.

- Discuss the spinner outcomes. Ask students what they can conclude from their experiment. They should identify the fact that even if their spinner meets the requirements, the outcomes may not match the probability statements. Have them compare spinners to identify how design differences might affect the outcomes.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students produced complete sample spaces; clearly demonstrated the ability to identify and compare theoretical and experimental probabilities; clearly demonstrated the ability to create a spinner with parts that correspond to specific probability statements; clearly demonstrated the ability, when needed, to alter a spinner to more closely correspond to specific probability statements; and clearly demonstrated the ability to communicate and explain mathematical ideas relating to probability using appropriate vocabulary and concepts.
- **2 points:** Students produced sample spaces that were at least 80% complete; demonstrated the ability to identify and compare theoretical and experimental probabilities at least 80% of the time; demonstrated the ability to create a spinner with parts that somewhat correspond to specific probability statements; demonstrated the ability, when needed, to alter a spinner to more closely correspond to specific probability statements; and demonstrated the ability to communicate and explain mathematical ideas relating to probability using some appropriate vocabulary and concepts.
- **1 point:** Students produced sample spaces that were less than 80% complete; demonstrated the ability to identify and compare theoretical and experimental probabilities less than 80% of the time; did not demonstrate the ability to create a spinner with parts that correspond to specific probability statements; did not demonstrate the ability to alter a spinner to more closely correspond to specific probability statements; and did not demonstrate the ability to communicate and explain mathematical ideas relating to probability using appropriate vocabulary and concepts.

Vocabulary

experimental probability

Definition: A probability based on the results of repeated trials of an experiment

Context: If your school bus is on time 32 out of 45 times, the experimental probability that the

bus is on time is $\frac{32}{45}$.

outcome

Definition: The possible results when an experiment is performed

Context: The possible outcomes when flipping a coin are heads or tails.

probability

Definition: A number from zero to one that expresses the likelihood that a specific event or outcome will occur

Context: Ula put 2 red chips, 4 blue chips, and 8 green chips in a bag. The probability of choosing a red chip is $\frac{2}{14}$, or $\frac{1}{7}$.

sample space

Definition: A list of all possible outcomes of a certain event

Context: Ken created a sample space in order to understand the combinations that could occur from rolling a number cube and flipping a coin.

theoretical probability

Definition: When all outcomes are equally likely, the theoretical probability of an event is the ratio of the number of favorable outcomes to the number of possible outcomes

Context: If bag of 10 marbles contains 4 blue marbles, the theoretical probability of choosing a blue marble is $\frac{4}{10}$, or $\frac{2}{5}$.

Academic Standards

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit

<http://www.mcrel.org/compendium/browse.asp>.

This lesson plan addresses the following benchmarks:

- Determines probability using mathematical/theoretical models (e.g., table or tree diagram, area model, list, sample space).
- Determines probability using simulations or experiments.
- Understands how predictions are based on data and probabilities (e.g., the difference between predictions based on theoretical probability and experimental probability).
- Understands that the measure of certainty in a given situation depends on a number of factors (e.g., amount of data collected, what is known about the situation, how current data are).
- Understands the relationship between the numerical expression of a probability (e.g., fraction, percentage, odds) and the events that produce these numbers.

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to

<http://standards.nctm.org>.

This lesson plan addresses the following standards:

- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.
 - Understand and use ratios and proportions to represent quantitative relationships.
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Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- <http://school.discovery.com/teachingtools/teachingtools.html>
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DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How to Use the DVD

The DVD starting screen has the following options:

Play Video – This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index – Here the video is divided into chapters indicated by title. Each chapter is then divided into four sections indicated by video thumbnail icons; brief descriptions are noted for each section. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Quiz – Each chapter has four interactive quiz questions correlated to each of the chapter's four sections.

Standards Link – Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources – This screen gives the technical support number and Web site address.

Video Index

I. Determining Probability (10 min.)

Determining Probability: Introduction

Take a look at probabilities expressed as fractions — the numerator is the number of ways an outcome can happen and the denominator is the total number of possible outcomes.

Example 1: Probability Tables

See how creating a sample space helps determine the probability of a specific outcome.

Example 2: Multiple Events

Learn how to calculate the probability of multiples events. Equally likely events are defined as events that have the same chance of occurring.

Example 3: Coin Tosses

Find out why the probability of tossing heads is always $\frac{1}{2}$ and take a closer look at determining the probability of multiple events using a chart.

II. Determining Probability Empirically (10 min.)

Determining Probability Empirically: Introduction

Learn how probability is determined through observation and experimentation over time.

Example 1: Historical Probabilities

Investigate how historical data can be used to predict the probability of future events as a basketball player uses his success ratio to predict the number of free throws he will during a game.

Example 2: Experimental Probability

Probabilities can be determined based on past events and experimentation. Watch as the probability of producing defective bikes is calculated based on past production data.

Example 3: Modeling Probabilities

See meteorologists use computer models, observations, and past weather patterns to predict the weather and report their predictions using probabilities.

III. Bases of Predictions (10 min.)

Bases of Predictions: Introduction

Compare theoretical probability, the chance an event will occur under defined circumstances, and experimental probability, the chance an event will occur based on the number of possible opportunities.

Example 1: Theoretical Probability

Take a closer look at theoretical probabilities as they are used to determine the probability of a family having three children all the same gender.

Example 2: Probability Experiment

Investigate experimental probability by calculating the probability of the number of boys or girls born using repeated observation.

Example 3: Experimental Probability

Discover how a batting average is an example of experimental probability.

IV. Measuring Probability (10 min.)

Measuring Probability: Introduction

See how calculating the probability an event will happen is based on the amount of data available and the reliability and accuracy of the data.

Example 1: Amount of Data

Take a look at the types of data seismologists gather to predict earthquakes and how they make more accurate and reliable predictions as the amount of data increases.

Example 2: Information and Probability

Watch glider pilots gather data on altitude, temperature, wind speed and direction, and location to predict the best time for a successful flight.

Example 3: Currency of Data

See why meteorologists rely on technology and up-to-the-minute data on temperature, humidity, air pressure, and wind speed to make weather forecasts.

V. Probability and Events (9 min.)

Probability and Events: Introduction

Explore how probabilities can be expressed as fractions, ratios, or percents and odds are expressed as ratios between chosen outcomes and remaining alternative outcomes.

Example 1: Probability Expressed as Fractions

Learn how to express probability as a fraction where the numerator reflects the number of favorable outcomes and the denominator reflects the total number of outcomes.

Example 2: Probabilities Expressed as Percents

See how to convert fraction probabilities to percents.

Example 3: Odds

Compare odds and probabilities and see how odds are expressed as a ratio between the chosen outcomes and the remaining alternative outcomes.

Quiz

I. Determining Probability

1. Ned puts ten yellow, seven red, and nine green marbles in a bag and then chooses one. What is the probability the marble is green?

A. $\frac{9}{26}$

B. $\frac{9}{17}$

C. $\frac{1}{3}$

D. 1

Answer: A

2. The chart shows the possible outcomes of the gender of a baby (XX = girl XY = boy). What outcome belongs in the empty box in the sample space?

A. XX

B. XY

C. YY

D. XYX

	X	X
X	XX	XX
Y	XY	

Answer: B

3. Paul has two 1-6 number cubes. If he rolls both cubes, what is the probability that the sum of the numbers on the cubes will equal 5?

A. $\frac{4}{9}$

B. $\frac{5}{36}$

C. $\frac{1}{9}$

D. $\frac{1}{18}$

Answer: C

4. What is the combined probability of a coin landing on tails four times in a row?

A. $\frac{1}{16}$

B. $\frac{4}{16}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer: A

II. Determining Probability Empirically

1. Walter's success ratio for free throws is $\frac{3}{4}$ or 75%. If he attempted 16 free throws during one game, based on his success ratio, how many free throws probably went in the hoop?

A. 3

B. 6

C. 12

D. 16

Answer: C

2. Last year Bob's Bikes produced 10,000 bikes. Of these, 20 bikes were defective. Based on the experimental probability, how many bikes will be defective out of the next 1,000 bikes produced?

A. 1

B. 2

C. 10

D. 20

Answer: B

3. A meteorologist notices rain clouds forming on the computer weather model. In the past the probability of rain was 80% when these clouds formed. Based on this information, what weather will the meteorologist probably predict?

A. clear skies with cold temperatures

B. cloudy with a small chance of rain

C. cloudy with a high chance of rain

D. clear skies with no rain

Answer: C

III. Bases of Predictions

1. What is the theoretical probability that a family will have five children of the same gender?

- A. $\frac{1}{5}$
- B. $\frac{1}{2}$
- C. $\frac{1}{32}$
- D. $\frac{1}{16}$

Answer: D

2. The theoretical probability of having a baby girl is $\frac{1}{2}$.

If in one day 9 baby girls and 12 baby boys are born at the local hospital, which statement is true?

- A. The theoretical probability of having a girl is greater than the experimental probability.
- B. The experimental probability of having a girl is greater than the theoretical probability.
- C. The theoretical probability of having a girl is less than the experimental probability.
- D. The theoretical probability of having a girl is equal to the experimental probability.

Answer: A

3. A baseball player has gotten a hit 19 out of the 75 times he has been at bat this season. Today he gets 2 hits out of the 4 times he is at bat. What is his batting average?

- A. 0.253
- B. 0.266
- C. 0.273
- D. 0.280

Answer: B

IV. Measuring Probability

1. Identify the true statement.
 - A. The amount of data a seismologist analyzes has no impact on the accuracy and reliability of her prediction.
 - B. As the amount of data a seismologist has to analyze increases her predictions become more accurate and reliable.
 - C. As the amount of data a seismologist has to analyze decreases her predictions become more accurate and reliable.
 - D. As the amount of data a seismologist has to analyze increases her predictions become less accurate and reliable.

Answer: B

2. Identify the most effective way for meteorologists to predict weather.
 - A. Collect important data once a day using technology.
 - B. Collect important data once in the morning and once in the evening.
 - C. Continually collect data throughout the day using weather balloons.
 - D. Continually collect data throughout the day using weather tools and technology.

Answer: D

V. Probability and Events

1. The probability of choosing a red ball out of a bag is $\frac{1}{4}$.

What are the odds of picking the red ball out of the bag?

- A. 1:4
- B. 4:1
- C. 1:3
- D. 3:4

Answer: C

2. The following shape cards are cut out and placed in a bag. What is the probability of choosing a heart card?

- A. $\frac{1}{5}$
- B. $\frac{1}{4}$
- C. $\frac{4}{5}$
- D. 1

♦	↑	↑	♥	♦
♦	♥	•	♦	↑
♦	↑	♦	♥	♦
♦	♥	•	♦	↑

Answer: A

3. The following shape cards are cut out and placed in a bag. What are the odds of choosing an arrow card?

- A. 1:3
- B. 1:4
- C. 1:1
- D. 3:5

Answer: A

♦	↑	↑	♥	♦
♦	♥	•	♦	↑
♦	↑	♦	♥	♦
♦	♥	•	♦	↑

4. There are 20 mystery books, 15 adventure books, 5 historical fiction books, and 10 comedy books in the classroom library. What is the probability that Mark will randomly choose an adventure book out of the library?

- A. 15%
- B. 30%
- C. 42%
- D. 70%

Answer: B

Theoretical and Experimental Probability Worksheet

Create the sample space to show the outcomes of choosing one cube from each bag.

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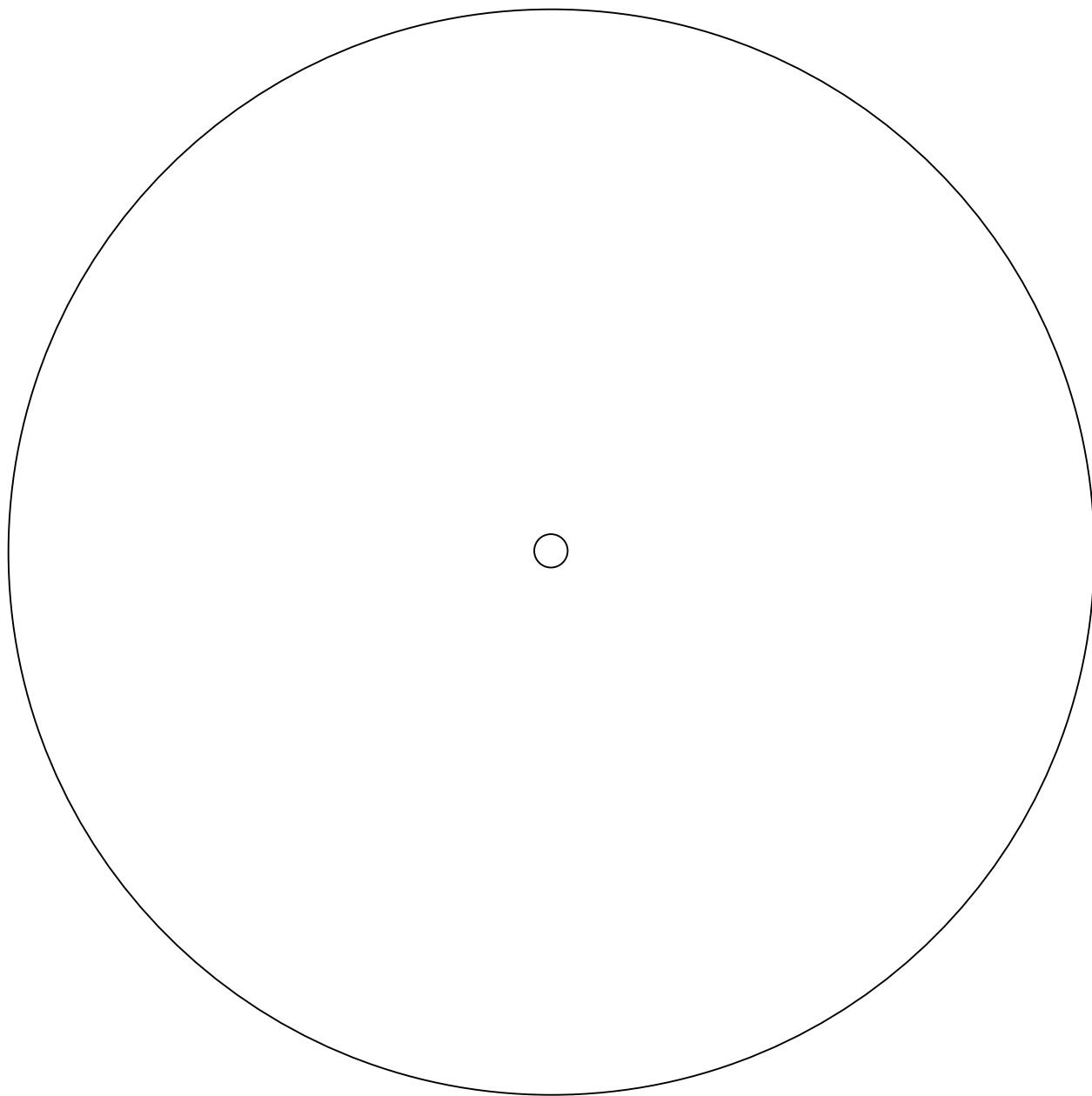
Use the sample space to complete the chart.

Outcome	Probability
Red, Red	
Red, Green	
Red, Blue	
Green, Green	
Green, Blue	
Blue, Blue	

Outcome	Theoretical Probability	Tally	Experimental Probability	Conclusion
Red, Red				
Red, Green				
Red, Blue				
Green, Green				
Green, Blue				
Blue, Blue				

Spinner Template

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Probability Statements Worksheet

1. The probability of landing on blue is greater than the probability of landing on red.
2. The probability of landing on white is less than the probability of landing on red.
3. The probability of landing on white is less than the probability of landing on green.
4. The probability of landing on red is equal to the probability of landing on green.
5. The probability of landing on yellow is greater than the probability of landing on white.
6. The probability of landing on green is greater than the probability of landing on yellow.
7. The probability of landing on blue is greater than the probability of landing on yellow.
8. The probability of landing on red is greater than the probability of landing on yellow.

Use the probability statements to create a spinner. Spin the spinner, record the outcome, and repeat 50 times. Calculate the experimental probability of each outcome in the chart.

Outcome	Tally	Experimental Probability
Red		
Yellow		
Green		
Blue		
White		

Compare the experimental probabilities to the probability statements. What conclusions can you make about your spinner and the experimental probabilities?