

Discovering Math: Expanding Problem-Solving Skills

Teacher's Guide

Grade Level: 3–5

Curriculum Focus: Mathematics

Lesson Duration: Three class periods

Program Description

Discovering Math: Expanding Problem-Solving Skills – From diagrams to trial-and-error to valid and invalid arguments, introduce elementary students to more-advanced strategies for solving problems.

Lesson Plan

Student Objectives

- Use a variety of strategies to understand and solve a problem in a team setting (e.g., discussing with peers, stating problems in own words, modeling problem with diagrams or physical objects, identifying a pattern).
- Distinguish between pertinent and irrelevant information when solving problems.
- Use the basic language of logic (e.g., and, or, not) when describing a solution to a problem.

Materials

- *Discovering Math: Expanding Problem-Solving Skills* video
- 1 box of spaghetti for each team
- 30 straws for each team
- 30 popsicle sticks for each team
- Tape
- Glue
- 3" x 12" piece of cardboard for each team
- Modeling clay for each team
- String
- 1 box of toothpicks for each team
- Problem-Solving Record Sheet (see below)
- 5 one-pound weights

Procedures

1. Present the problem-solving situation.

With the given materials, build a bridge that can support at least five pounds. Tell students they will work in teams, use some or all the materials in the bag, and use the problem-solving strategies to create a bridge.

Review the problem-solving strategies presented in the video.

- discussion
- pictures and diagrams
- making lists
- logical reasoning
- working backwards
- making guesses and tests at solutions

Discuss each strategy and its benefits. Emphasize the importance of teamwork.

2. Divide the class into teams. Give each team a bag of materials (1 box of spaghetti, 30 straws, 30 popsicle sticks, tape glue, cardboard, modeling clay, string, and toothpicks). Give each student a Problem-Solving Record Sheet.
3. Have the teams discuss the presented problem. They should discuss all facets of the problem, including the materials provided. Remind them to work backwards and identify the ultimate goal of the bridge.
4. Ask each student to develop several possible solutions and record them on their record sheet.
 - They should draw diagrams of their ideas. The diagrams must be labeled to show the specific materials that will be used.
 - They should create a list of the possible solutions. It should contain the idea, a statement indicating why they think the idea will work, and the materials needed.

If needed, model one solution for the class so they have guide to follow.

5. Each team should now discuss the possible solutions and select one to pursue.
6. Display the term logic statement. Model logic statements using and, or, and not. Have students write two logic statements relating to the problem (e.g., Materials used to build bridges are strong *and* sturdy). Depending on ability levels, some students may need assistance with logic statements.
7. Have each team create a model of their bridge. Remind them to use only the materials provided. Allow an appropriate amount of time to create the model, to be determined based on the students' abilities.
8. When all bridges are complete test their strength. Place five one-pound weights on each bridge, one weight at a time. Teams have successfully solved the problem if their bridge supports five pounds.

9. Discuss the benefits of working in a team and implementing specific problem-solving strategies. Ask students if they can make any conclusions about bridge construction based on the solutions developed by each team. Have the teams brainstorm conclusions, record them on their record sheet, and share with the class. If possible, have students write valid and invalid statements about the bridge construction. Students may need a review of valid and invalid statements. Refer to examples from the video for models.

Extension:

Have students research effective bridge designs and construction after building their model. Ask them to use this information to develop new solutions or strengthen their existing bridge.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class and team discussions; produced complete Problem-Solving Record Sheets, including all the requested information; clearly demonstrated the ability to work in a team to solve a problem; and clearly demonstrated the ability to use specific problem-solving strategies.
- **2 points:** Students participated in class and team discussions; produced an adequate Problem-Solving Record Sheet, including most of the requested information; satisfactorily demonstrated the ability to work in a team to solve a problem; and satisfactorily demonstrated the ability to use specific problem-solving strategies.
- **1 point:** Students participated minimally in class and team discussions; created an incomplete Problem-Solving Record Sheet with little or none of the requested information; did not demonstrate the ability to work in a team to solve a problem; and did not demonstrate the ability to use specific problem-solving strategies.

Vocabulary

problem solving

Definition: a systematic method of examining a problem, developing possible solutions, and solving the problem

Context: The teacher gave each team the challenge of developing a new game. The teams used problem solving to think about the challenge, list possible solutions, and choose one solution.

Academic Standards

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit <http://www.mcrel.org/compendium/browse.asp>.

This lesson plan addresses the following benchmarks:

- Uses a variety of strategies to understand problem situations (e.g., discussing with peers, stating problems in own words, modeling problem with diagrams or physical objects, identifying a pattern)
- Uses trial and error and the process of elimination to solve problems
- Knows the difference between pertinent and irrelevant information when solving problems
- Understands the basic language of logic in mathematical situations (e.g., "and," "or," "not")

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to <http://standards.nctm.org>.

This lesson plan addresses the following standards:

- Apply and adapt a variety of appropriate strategies to solve problems
 - Monitor and reflect on the process of mathematical problem solving
 - Recognize reasoning and proof as fundamental aspects of mathematics
 - Develop and evaluate mathematical arguments and proofs
 - Recognize and use connections among mathematical ideas
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Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the DiscoverySchool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- <http://school.discovery.com/teachingtools/teachingtools.html>
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DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How to Use the DVD

The DVD starting screen has the following options:

Play Video – This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index – Here the video is divided into chapters indicated by title. Each chapter is then divided into four sections indicated by video thumbnail icons; brief descriptions are noted for each section. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Quiz – Each chapter has four interactive quiz questions correlated to each of the chapter's four sections.

Standards Link – Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources – This screen gives the technical support number and Web site address.

Video Index

I. Problem-Solving Strategies (6 min.)

Understanding How to Represent Problems: Introduction

Students use discussion, pictures, models, lists, logical reasoning, working backwards, and making guesses to solve problems.

Example 1: Discussing, Reasoning, and Modeling for Wing Design

Students use discussion, drawing, and logical reasoning to make a copper model of the Wright brothers' first wing design.

Example 2: Logical Reasoning and Measurement for Fuel Use

Students use logical reasoning, measuring, and calculating to determine which of five fuels will power a cart to travel the farthest distance.

Example 3: Comparing Strategies

Students collect and analyze data to test the quality of the water in a stream. They find that the higher the biodiversity, the higher the quality of water.

II. Trial-and-Error Problem Solving (6 min.)

Trial-and-Error Problem Solving: Introduction

Paleontologists use trial-and-error problem solving to search for dinosaur fossils.

Example 1: Elimination of Unusable Material

People in Alaska use trial-and-error to solve problems when building igloos. Trial-and-error helps eliminate inadequate solutions in real-world problems.

Example 2: Testing Solutions

Engineers use trial-and-error problem solving to identify the strongest materials to use in house foundations.

Example 3: Guess-and-Check Calculation

People use trial-and-error problem solving to solve real-life problems such as determining the cost of a meal.

III. Information Relevance (8 min.)

Information Relevance: Introduction

Scientists use relevant information to theorize that an asteroid hit the Earth and caused the extinction of dinosaurs.

Example 1: Relevant Information

A woman uses relevant information—the amount of quarters it takes to do laundry—to determine how many loads of laundry she can do.

Example 2: Information Relevance

A boy uses relevant information to determine if learning a new dive is going to improve his diving score and allow him to win the competition.

Example 3: Goal-Related Relevance

Information is deemed relevant based on the goal or specific problem a person is trying to achieve or solve. Weather is relevant information to all marathoners, regardless of their goal.

IV. Language of Logic (6 min.)

Language of Logic: Introduction

Kids in a competition use logic and logic statements to solve the problem of which two teams to eliminate to improve their chances of winning.

Example 1: "And" Statements

Students use logic statements to determine animals that have the same characteristics. Logic statements using "and" add more requirements. All factors must be true for the whole statement to be true.

Example 2: "Or" Statements

The word "or" is used in logic statements to show that one of the requirements being considered must be true. "Or" is used when classifying birds and their ability to run fast, swim, or fly.

Example 3: "And," "Or," and "Not" Statements

Logic helps people solve problems and make decisions. The campers use logic and logic statements with the words "and," "or," and "not" to divide themselves into 4 teams.

V. Reasonable Solutions (9 min.)

Reasonable Solutions: Introduction

The Hubble Space Telescope needed repairs. Scientists discussed several reasonable solutions for fixing it. They analyzed potential solutions and outcomes with reasoning and logic.

Example 1: Determining a Solution

Scientists developed several solutions to fix the flawed optics on the Hubble Space Telescope. They analyzed the reasonableness and feasibility of each solution before deciding to have astronauts place a corrective lens on the damaged mirror.

Example 2: Timing a Solution

The gyroscopes on the Hubble Space Telescope needed repair, but the shuttle launch was delayed due to bad weather. Why was postponing the launch the most reasonable solution?

Example 3: Selecting a Solution

Four of the six gyroscopes on the Hubble Space Telescope needed to be replaced. Scientists and engineers determined the most reasonable and feasible solution was to send astronauts to replace the gyroscopes.

VI. Valid and Invalid Arguments (6 min.)

Valid and Invalid Arguments: Introduction

In math an argument can be a set of statements with logical reasoning behind them. Valid arguments lead to true conclusions and invalid arguments lead to untrue conclusions.

Example 1: Counterexamples

A counterexample provides a valid argument to disprove a statement. Counterexamples show that Columbus did not discover America.

Example 2: Irrelevant Approaches

Irrelevant arguments are invalid, even if the information is correct. An argument about dogs, astronauts, and weightlessness is used to model irrelevant arguments.

Example 3: Invalid Reasoning

True statements about sharks and lightning are given. The likelihood of an event can be affected by behavior.

Quiz

I. Problem-Solving Strategies

1. What problem-solving strategy allows team members to learn from each other and understand the problem from different perspectives?
A. discussion
B. making a list
C. making models
D. logical reasoning

Answer: A

2. What was the goal in the jet fuel challenge?
A. to determine which jet flies the fastest
B. to determine which fuel burns the fastest
C. to determine which model airplane can fly the highest
D. to determine which fuel makes a cart travel the farthest

Answer: D

3. Identify the correct relationship between biodiversity and water quality.
A. the lower the biodiversity, the higher the water quality
B. the higher the biodiversity, the lower the water quality
C. the higher the biodiversity, the higher the water quality
D. there is no relationship between biodiversity and water quality

Answer: C

II. Trial-and-Error Problem Solving

1. What problem-solving strategy are people using when they make a guess at a solution, test their guess, and then adjust their guess based on the results?
A. discussion
B. trial and error
C. make a model
D. logical reasoning

Answer: B

2. Eskimos use trial-and-error problem solving to eliminate inadequate solutions when _____.
A. fishing
B. hunting
C. building igloos
D. storing ice blocks

Answer: C

3. The trial-and-error strategy develops and tests techniques that will be used later. Engineers and builders use trial-and-error problem solving to identify ____.
- A. how long it will take to build a house
 - B. the best shape and weight of cement blocks
 - C. the strongest materials for the roof of a house
 - D. the strongest materials for the foundation of a house

Answer: D

4. Four people go out to dinner and decide to split the check evenly. The bill, including tip, is \$102.00. How much does each person have to pay?
- A. \$20.50
 - B. \$25.00
 - C. \$25.50
 - D. \$30.00

Answer: C

III. Relevant Information

1. Scientists found uncommon mineral elements in the Earth. What did the scientists conclude about dinosaurs from this information?
- A. Dinosaurs ate different minerals.
 - B. Asteroids caused dinosaurs to change their diet.
 - C. An asteroid hit the Earth and caused the extinction of dinosaurs.
 - D. Dinosaurs lived in different locations after an asteroid hit the Earth.

Answer: C

2. You and your friends would like to buy drinks from a vending machine. The drinks cost \$.75 each and the machine only takes quarters. You have \$3.00 — 10 quarters and 5 dimes. How many drinks can you buy?
- A. 2
 - B. 3
 - C. 4
 - D. 5

Answer: B

3. Tom and his coach decide that he should learn the arm-stand dive to increase his diving score. What two skills does Tom need to work on to perform the arm-stand dive well?
- A. balance and leg strength
 - B. arm strength and balance
 - C. arm strength and leg strength
 - D. jumping off the diving board and arm strength

Answer: B

4. What information is relevant to all marathoners, regardless of their goal?
 - A. the weather
 - B. how fast they run
 - C. how fast others run
 - D. how long it takes them to finish the marathon

Answer: A

IV. Language of Logic

1. Five teams are competing for a prize. The red team is the strongest; the green team is the weakest; and the blue, purple, and yellow teams are in the middle. The green team gets to eliminate one team from the competition. Using logic, which team should the green team eliminate to improve its chances of winning?

- A. the red team
- B. the blue team
- C. the purple team
- D. the yellow team

Answer: A

2. The children studied types of transportation: cars, bikes, motorcycles, and airplanes. The teacher asked them to name the type of transportation that runs on gas **and** has windows **and** flies in the air. What type of transportation fits the description?

- A. car
- B. bike
- C. airplane
- D. motorcycle

Answer: C

3. What word is used in logic statements to show that one of the requirements being considered must be true? For example, a penguin can swim, but it can't fly ____ run fast.

- A. or
- B. not
- C. and
- D. although

Answer: A

4. Sue, Mark, Lou, and Brit are divided into two teams. Each team has two players only. Which logic statement is true?

- A. Mark or Lou or Sue are on team 1.
- B. Brit and Mark are not on a team.
- C. Mark and Lou and Brit are on team 1.
- D. and Sue are on team 1 and team 2.

Answer: A

V. Reasonable Solutions

1. Identify the solution scientists determined was most reasonable to fix the flawed optics on the Hubble Space Telescope.
 - A. send robots
 - B. clean the lens
 - C. corrective lens
 - D. replace the mirror

Answer: C

2. What problem caused the shuttle launch to be postponed?
 - A. weather
 - B. wed optics
 - C. enough fuel
 - D. failing gyroscopes

Answer: A

3. What solution did the scientists and engineers determine was the most reasonable and feasible to fix the damaged gyroscopes on the Hubble Space Telescope?
 - A. send robots to repair the gyroscopes
 - B. send astronauts to replace the gyroscopes
 - C. send robots and astronauts to repair the gyroscopes
 - D. send astronauts to put a corrective lens on the gyroscopes

Answer: B

VI. Valid and Invalid Arguments

1. All dogs have four legs. All poodles are dogs. Therefore, _____.
What is the true conclusion to of this valid argument?
 - A. all dogs are poodles
 - B. all dogs have four legs
 - C. all poodles have four legs
 - D. all four legged animals are dogs

Answer: C

2. What is an example that shows an idea is wrong or provides a valid argument to disprove a statement?
 - A. valid example
 - B. counterexample
 - C. invalid argument
 - D. counter argument

Answer: B

3. Bea provides the following argument:
Astronauts are weightless in space.
Mice are not astronauts.
Therefore, mice are not weightless in space.
Since everything is weightless in space, what type of argument is this?
- A. valid argument
 - B. invalid statements
 - C. incorrect argument
 - D. irrelevant argument

Answer: D

4. If you are under a tree during a thunderstorm you are more likely to get struck by lightning. What can affect the likelihood of this event occurring?
- A. when the thunderstorm occurs
 - B. how many times you go swimming
 - C. your behavior during thunderstorms
 - D. the type of clothes you wear during a thunderstorm

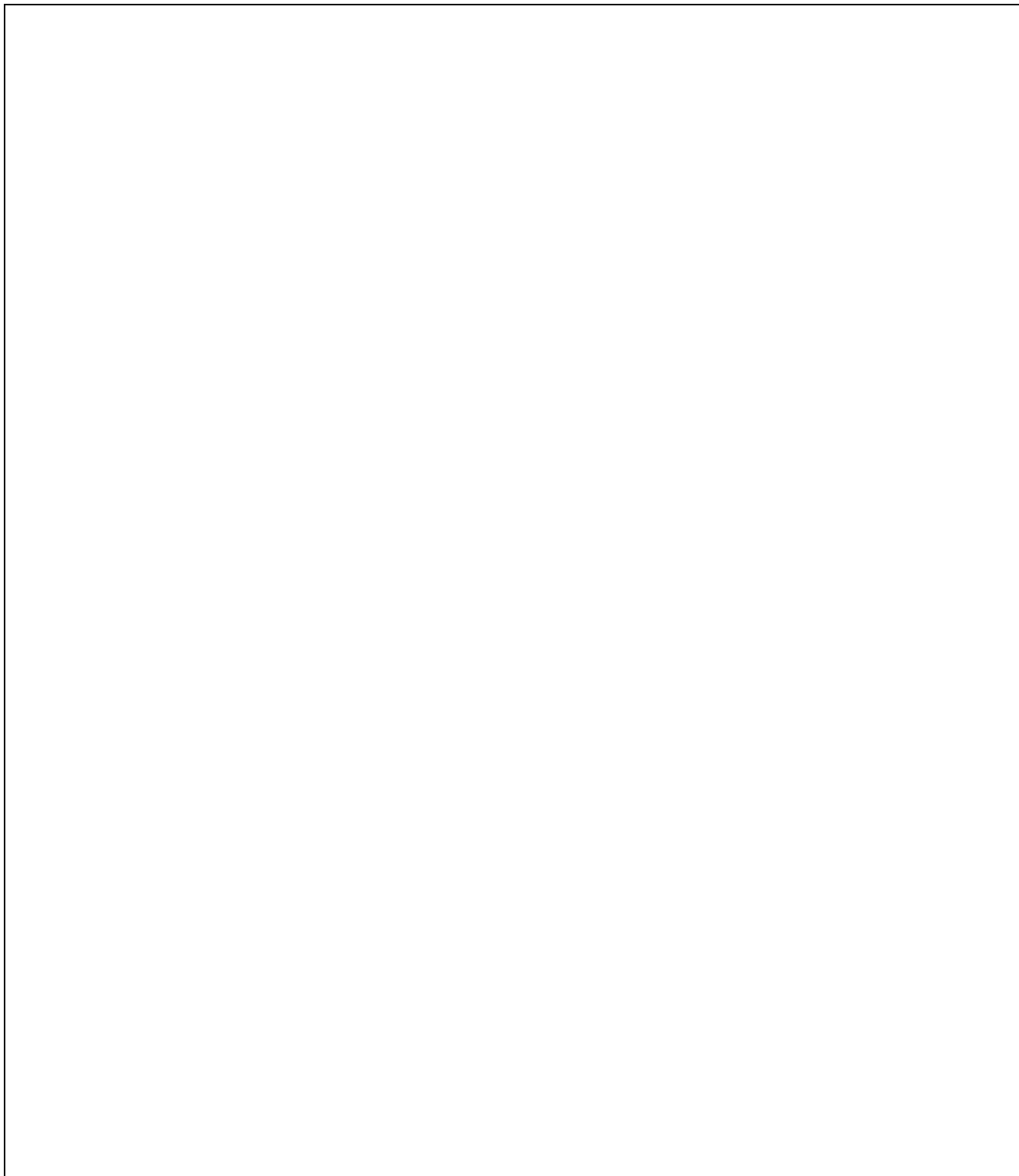
Answer: C

Problem-Solving Record Sheet

Team Members

Possible Solutions

Solution Diagrams



Possible Solution	Idea-supporting Statement	Materials Needed

Solution

Logic Statements

Conclusions
