

Discovering Math: Concepts in Algebra

Teacher's Guide

Grade Level: 3–5

Curriculum Focus: Mathematics

Lesson Duration: Three class periods

Program Description

Discovering Math: Concepts in Algebra — From representing patterns to understanding the concepts of variables and equal relationships, introduce elementary students to more advanced properties of functions and algebra.

Lesson Plan

Student Objectives

- Recognize and create a variety of patterns.
- Understand that a pattern can be represented in a variety of ways.
- Demonstrate an understanding of variables and their uses.
- Solve open sentences involving operations on whole numbers.
- Demonstrate an understanding of and the ability to use a rectangular coordinate grid system.

Materials

- *Discovering Math: Concepts in Algebra* video
- Calculators
- Open Sentence Practice Sheets (see below)
- Rectangular Coordinate Grids (see below)

Procedures

1. Display the word pattern. Have students work with a partner to discuss the word and create an example. A pattern is an ordered set of numbers or objects. The examples should demonstrate an understanding of patterns. Have each pair display and explain their pattern.

Ask students to recall the types of patterns presented in the video. They should list repeating, increasing, and decreasing patterns.

- Model a repeating pattern. Display the following:

4, 4, 3, 3, 2, 2, 4, 4, 3, 3, 2, 2,...

Ask students to identify the pattern and rule (2 fours, 2 threes, and 2 twos). Then ask them to extend the pattern.

- Model an increasing pattern. Display the following:

1, 4, 7, 10, 13,...

Ask students to identify the pattern and rule (add 3). Then ask them to extend the pattern.

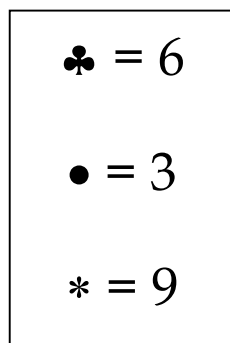
- Model a decreasing pattern. Display the following:

974, 964, 954, 944,...

Ask the students to identify the pattern and rule (subtract 10). Then ask them to extend the pattern.

- Ask each student to create a repeating, increasing, and decreasing pattern. Have them switch patterns with a partner. The partners should identify each pattern and the rule. Then they should extend the patterns.

2. Display the following key:



Tell students that numeric patterns can be represented with symbols. Model a pattern using the symbols from the key.

●♣♣●♣♣

Ask students to use the key to translate the pattern into numbers.

9, 9, 9, 3, 6, 6, 9, 9, 9, 3, 6, 6

Ask students to create their own patterns using symbols. They should also create a key for their patterns. Have them switch patterns with their partner and translate and extend the patterns.

3. Pose the following problem to students:

Mary needs \$10 to buy a new calculator, but she only has \$6. How much more money does Mary need to buy the calculator?

Allow time to discuss and devise a way to solve the problem. Have students share their ideas.

Show students how to write an open sentence for the problem.

$$6 + ? = 10$$

Explain that they can use a letter or symbol to represent the unknown. Label the letter or symbol as the variable.

Review inverse operations with students (addition and subtraction, multiplication and division). Note: For younger students use only addition and subtraction.

Model how to use inverse operations to solve the open sentence (subtract 6 from both sides so ? equals 4).

- Assign each student a partner. Distribute copies of the Open Sentence Practice Sheets to each pair. Allow time to solve each open sentence. Students should show their work and be able to explain how they solved each problem. Remind them that the equal sign in a number sentence shows the values on each side are equal and the same computation must be done on each side to maintain equality. Allow students to check their work with a calculator if needed.
 - When students have completed the practice ask them to write their own open sentences to challenge their partners.
 - Extension: Challenge students to solve open sentences with more than one step (e.g., $4 + 3 + ? = 20$). They can write their own challenge problems and share with a partner.
4. Display a rectangular coordinate grid. Review how to identify an ordered pair that names a point on the grid (students' ability levels will determine if an one-quadrant or four-quadrant grid is used). Allow students time to practice identifying points on a grid and giving the ordered pair.
- Distribute copies of the grids to students. Have them place the letters of the alphabet as points on the grid (be sure the points are on intersections so they can be properly identified).
 - Tell students to write a 2-3 word secret message.
 - Have them write the ordered pairs to identify each letter in the message. Remind them to name the point on the x-axis first and the point on the y-axis second. Refer to the ordered pairs as the code needed to interpret the secret message.
 - Have students switch codes with a partner. They should use the ordered pairs to identify the secret message.

Assessment

Use the following three-point rubric to evaluate students' work during this lesson.

- **3 points:** Students were highly engaged in class discussions; clearly demonstrated the ability to identify, extend and create repeating, increasing, and decreasing patterns; clearly demonstrated the ability to solve open sentences; and clearly demonstrated the ability to identify and label points on a grid using ordered pairs.
- **2 points:** Students participated in class discussions; satisfactorily demonstrated the ability to identify, extend and create repeating, increasing, and decreasing patterns; satisfactorily demonstrated the ability to solve open sentences; and satisfactorily demonstrated the ability to identify and label points on a grid using ordered pairs.
- **1 point:** Students participated minimally in class discussions; did not demonstrate the ability to identify, extend and create repeating, increasing, and decreasing patterns; did not demonstrate the ability to solve open sentences; and did not demonstrate the ability to identify and label points on a grid using ordered pairs.

Vocabulary

ordered pair

Definition: a pair of numbers that names a point on a grid

Context: The ordered pair (4,5) names point P on the grid. P is located at 4 on the x-axis and 5 on the y-axis.

pattern

Definition: an ordered set of numbers or objects

Context: The teacher displayed the following numbers: 2, 4, 6, 8, 10, ... He asked the student to identify the pattern. They identified the pattern as counting by two.

rectangular coordinate grid

Definition: a grid based on two perpendicular axes (x-axis and y-axis)

Context: The ordered pair (4,5) names point P on the rectangular coordinate grid. The grid is based on two perpendicular axes. The first number in the ordered pair refers to the location on the x-axis and the second number refers to the location on the y-axis.

variable

Definition: symbols that represent unknown values or values that can change

Context: The variable in the number sentence $4 + 3 = ?$ is 7 because it is the unknown value.

Academic Standards

Mid-continent Research for Education and Learning (McREL)

McREL's Content Knowledge: A Compendium of Standards and Benchmarks for K-12 Education addresses 14 content areas. To view the standards and benchmarks, visit <http://www.mcrel.org/compendium/browse.asp>.

This lesson plan addresses the following benchmarks:

- Recognizes a wide variety of patterns and the rules that explain them.
- Understands that the same pattern can be represented in different ways.
- Knows that a variable is a letter or symbol that stands for one or more numbers.
- Understands the basic concept of an equality relationship.
- Solves simple open sentences involving operations on whole numbers.
- Knows basic characteristics and features of the rectangular coordinate system.

National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) has developed national standards to provide guidelines for teaching mathematics. To view the standards online, go to <http://standards.nctm.org>.

This lesson plan addresses the following standards:

- Describe, extend, and make generalizations about geometric and numeric patterns.
 - Represent and analyze patterns and functions, using words, tables, and graphs.
 - Represent the idea of a variable as an unknown quantity using a letter or a symbol.
 - Express mathematical relationships using equations.
 - Investigate how a change in one variable relates to a change in a second variable.
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Support Materials

Develop custom worksheets, educational puzzles, online quizzes, and more with the free teaching tools offered on the Discoveryschool.com Web site. Create and print support materials, or save them to a Custom Classroom account for future use. To learn more, visit

- <http://school.discovery.com/teachingtools/teachingtools.html>
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DVD Content

This program is available in an interactive DVD format. The following information and activities are specific to the DVD version.

How to Use the DVD

The DVD starting screen has the following options:

Play Video – This plays the video from start to finish. There are no programmed stops, except by using a remote control. With a computer, depending on the particular software player, a pause button is included with the other video controls.

Video Index – Here the video is divided into chapters indicated by title. Each chapter is then divided into four sections indicated by video thumbnail icons; brief descriptions are noted for each section. To play a particular segment, press Enter on the remote for TV playback; on a computer, click once to highlight a thumbnail and read the accompanying text description and click again to start the video.

Quiz – Each chapter has four interactive quiz questions correlated to each of the chapter's four sections.

Standards Link – Selecting this option displays a single screen that lists the national academic standards the video addresses.

Teacher Resources – This screen gives the technical support number and Web site address.

Video Index

I. Patterns and Rules (6 min.)

Patterns and Rules: Introduction

Patterns occur when an object or number changes or repeats itself in a systematic way. A variety of repeating patterns in nature are shown and explained.

Example 1: Recognizing Repeating Patterns

Ctenophores, a type of sea creature, are used to show repeating patterns in nature.

Example 2: Increasing Patterns

An exercise routine demonstrates an increasing number pattern – a type of repeating pattern in which the numbers or objects increase in a regular way.

Example 3: Decreasing Patterns

Leaves and trees demonstrate a decreasing pattern – a type of repeating pattern in which the numbers or objects decrease in a regular way.

II. Representing Patterns (8 min.)

Representing Patterns: Introduction

Crops represent numeric and geometric patterns. Once the rule for a pattern is identified, a variable equation can be used to determine the value of any row in the pattern.

Example 1: Simple Patterns

The rows of seats in a baseball stadium demonstrate how to represent the same pattern numerically and geometrically.

Example 2: Increasing and Decreasing Patterns

The Brooklyn Bridge is used to explain and model increasing and decreasing patterns that occur simultaneously in the same object.

Example 3: Geometric Arrays and Numeric Representations

Bowling pins are used to model and explain geometric arrays and numeric representations. Geometric patterns can be assigned numeric values. Variables can be used to calculate the value of any row.

III. Variables (8 min.)

Variables: Introduction

Symbols represent information that is widely understood and accepted. Variables are symbols that represent unknown values or values that can change.

Example 1: Unknown Quantities

A question mark or a letter can represent a variable in a number sentence. The unknown is another term for the variable.

Example 2: Variable Quantities

The change in bison and caribou populations is used to show variable quantities. Unknown quantities can be represented with $<$, $>$, and $=$ symbols, the variable, and the known amounts.

Example 3: Relationships Among Variables

When the relationship of two variables is known, a variable equation can be written to solve the problem. Determining numbers of cans of soup is used to model and explain a variable equation.

IV. Equality (9 min.)

Equality: Introduction

The equal sign in a number sentence shows that the values on each side are the same. The same computations must be done on each side of the equal sign to maintain equality.

Example 1: Rearranging Without Changing Quantities

The numbers or symbols on either side of the equal sign in an equation can be rearranged as long as the total quantities on each side remain the same.

Example 2: Changing Quantities, Keeping Equality

The numbers on one or both sides of an equation can change form without changing quantity. If the quantity on one side does change, the other side must change in the same way to maintain equality.

Example 3: Subtracting from Both Sides

Solving an equation with an unknown is modeled and explained. The values on each side of the equal sign can change, but the equality must stay the same.

V. Solving Open Sentences (9 min.)

Solving Open Sentences: Introduction

An open sentence is a statement in which some information is unknown. Use inverse operations and known information to solve for the unknown. Solving an open sentence is modeled and explained.

Example 1: Problems Involving Addition and Subtraction

A family uses open sentence equations to calculate how much money they must earn to meet their savings goal and how much extra money they have.

Example 2: Problems Involving Multiplication and Division

A girl uses open sentences and inverse operations to calculate how many hours she must work to meet her savings goal. Solving an open sentence with inverse operations is modeled and explained.

Example 3: Multi-Step Problems

A couple solves a multi-step problem to figure out how to meet their savings goal. A multi-step problem using inverse operations is modeled and explained.

VI. Rectangular Coordinate Systems (9 min.)

Rectangular Coordinate Systems: Introduction

A rectangular coordinate system is a grid based on two perpendicular axes. Ordered pairs name a point on the grid.

Example 1: Coordinate Mapping

Archaeologists use coordinate grids to mark their findings. The first number in the ordered pair refers to the x-axis coordinate. The second number of the ordered pair refers to the y-axis coordinate.

Example 2: Determining Coordinates

Determining the ordered pairs of spaces on a grid is modeled and explained. The x-axis coordinate is always listed first in the ordered pair.

Example 3: Plotting in All Quadrants

Scientists use coordinate grids to study ants. A four-quadrant grid with negative numbers on the x and y axes is introduced. Using ordered pairs on a grid is modeled and explained.

Quiz

I. Patterns and Rules

1. Laurie is making a repeating pattern of beads on her necklace. She puts two blues, three greens, two yellows, one red, two blues, three greens, and two yellows. What should Laurie use next to continue the repeating pattern?
A. one red
B. two blues
C. three greens
D. two yellows

Answer: A

2. Oliver is training for a race. He records how many miles he runs each week, and he notices a pattern. If the pattern continues, how many miles will Oliver run in week 5?
A. 23
B. 24
C. 27
D. 30

Week 1	Week 2	Week 3	Week 4	Week 5
12 miles	15 miles	18 miles	21 miles	

Answer: B

3. What number comes next in the pattern?
83, 79, 75, 71, 67, 63, _____
A. 59
B. 60
C. 66
D. 69

Answer: A

II. Representing Patterns

1. Farmer Brown plants seeds for 50 heads of lettuce in each row. What is the total number of heads of lettuce in rows one, two, and three?
A. 50
B. 100
C. 150
D. 200

Answer: C

2. Use the key to change the geometric pattern into a numeric pattern.

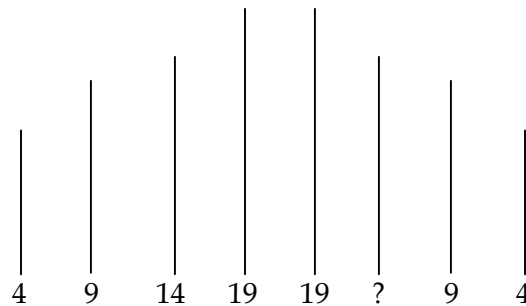
What numeric pattern is represented by the geometric pattern? $\blacklozenge\blacklozenge\blacklozenge\bullet\bullet\bullet\square\square$

- A. 3 2 9
B. 33 2 2 9 9
C. 333 2 2 2 9
D. 333 2 2 2 9 9
- $\blacklozenge = 3$
 $\bullet = 2$
 $\square = 9$

Answer: D

3. Look at the increasing and decreasing pattern.
Identify the correct number to complete the pattern.

- A. 4
B. 9
C. 14
D. 19



Answer: C

4. Let R = the row number
Let N = total number of pins through the end of the previous row

$R + N$ = total number of pins. If there is a total of 21 pins through the end of row 6, how many pins are there through the end of row 8?

- A. 14
B. 28
C. 35
D. 36

Answer: D

III. Variables

1. Jen has 35 books, Sue has 23, and Jill has 43. What is the total number of books the girls have? What is the variable in this situation?
- A. the number of books Jen has
B. the number of books Sue has
C. the number of books Jill has
D. the total number of books the girls have

Answer: D

2. There are 29 bananas in a bunch. A monkey takes some, so 18 bananas are left. If B represents the number of bananas taken, what number sentence represents this problem?
- A. $29 + B = 18$
 - B. $29 - B = 18$
 - C. $18 - B = 29$
 - D. $29 + 18 = B$

Answer: B

3. Lou researched the population of bald eagles. He recorded his data in a chart. If the number of pairs of bald eagles increased every year and E equals the number of pairs of bald eagles in 1990, identify the correct way to represent the number of pairs of bald eagles in 1990.
- A. $417 > E > 1,757$
 - B. $1,757 > E > 4,449$
 - C. $791 > E < 4,449$
 - D. $1,757 < E < 4,449$

Year	Number of Pairs of Bald Eagles
1963	417
1974	791
1984	1,757
1994	4,449

Answer: D

4. Linda has 10 cups, and each cup can hold 5 ounces of water. How many ounces of water does Linda need to fill all the cups?

Let C = number of cups

Let W = total ounces of water

Which variable equation can be used to solve the problem?

- A. $C + 5 = W$
- B. $W \times 5 = C$
- C. $C \times 5 = W$
- D. $C \times W = 5$

Answer: C

IV. Equality

1. $16 + 3 + 8 = 8 + \underline{\hspace{2cm}} + 16$
What number makes the number sentence true?
A. 3
B. 11
C. 24
D. 27

Answer: A

2. $26 = 26$
 $26 = \underline{\hspace{2cm}}$
A. $3 + 10$
B. $20 - 6$
C. $18 + 8$
D. $30 - 8$

Answer: C

3. $18 = 18$
 $18 \neq \underline{\hspace{2cm}}$
A. $10 + 8$
B. $20 - 3$
C. 3×6
D. $12 + 6$

Answer: B

4. Solve for X.
 $35 = 23 + X$
A. 12
B. 15
C. 22
D. 58

Answer: A

V. Solving Open Sentences

1. Solve for X.

$$X - 53 = 18$$

- A. 35
- B. 61
- C. 45
- D. 71

Answer: D

2. The Ludwig family wants to save \$500 for a weekend trip. They have already saved \$325. How much more money must they save?

$$X + \$325 = \$500$$

- A. \$150
- B. \$175
- C. \$225
- D. \$825

Answer: B

3. Amanda makes \$5 an hour baby-sitting. She needs \$120 to buy a plane ticket to visit her friend. How many hours must Amanda baby-sit to make enough money for the plane ticket?

- A. 10
- B. 14
- C. 24
- D. 26

Answer: C

4. Bob, Ed, and Ron work together mowing lawns. They charge \$35 to mow one lawn, and they share all the money equally. If they mow 24 lawns in one month, how much does each person make?

- A. \$70
- B. \$210
- C. \$280
- D. \$840

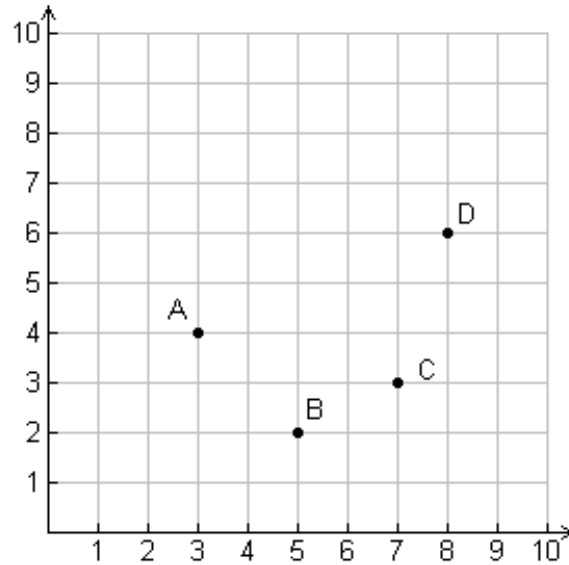
Answer: C

VI. Rectangular Coordinate System

1. Which point is located at $(7, 3)$?

- A. A
- B. B
- C. C
- D. D

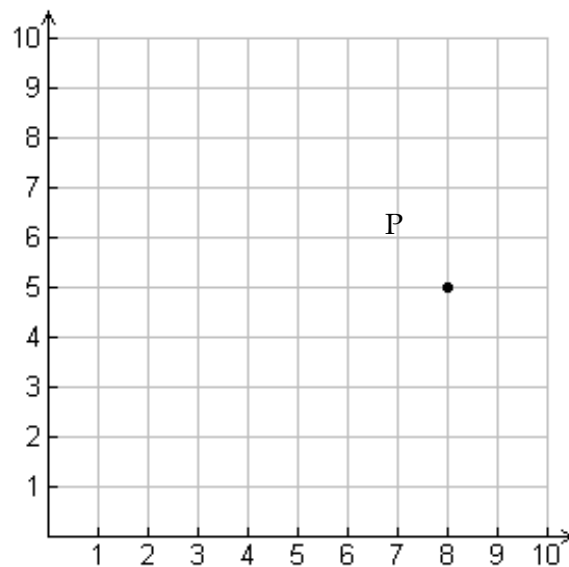
Answer: C



2. What is the ordered pair for point P?

- A. $(5, 8)$
- B. $(7, 5)$
- C. $(5, 9)$
- D. $(8, 5)$

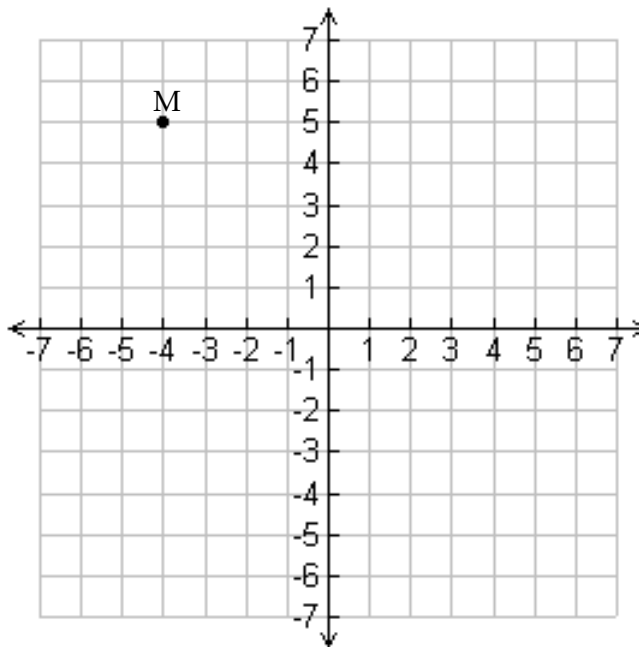
Answer: D



3. What is the ordered pair for point M?

- A. $(-4, 5)$
- B. $(5, -4)$
- C. $(4, 5)$
- D. $(-4, -5)$

Answer: A



Open Sentence Practice Sheet 1

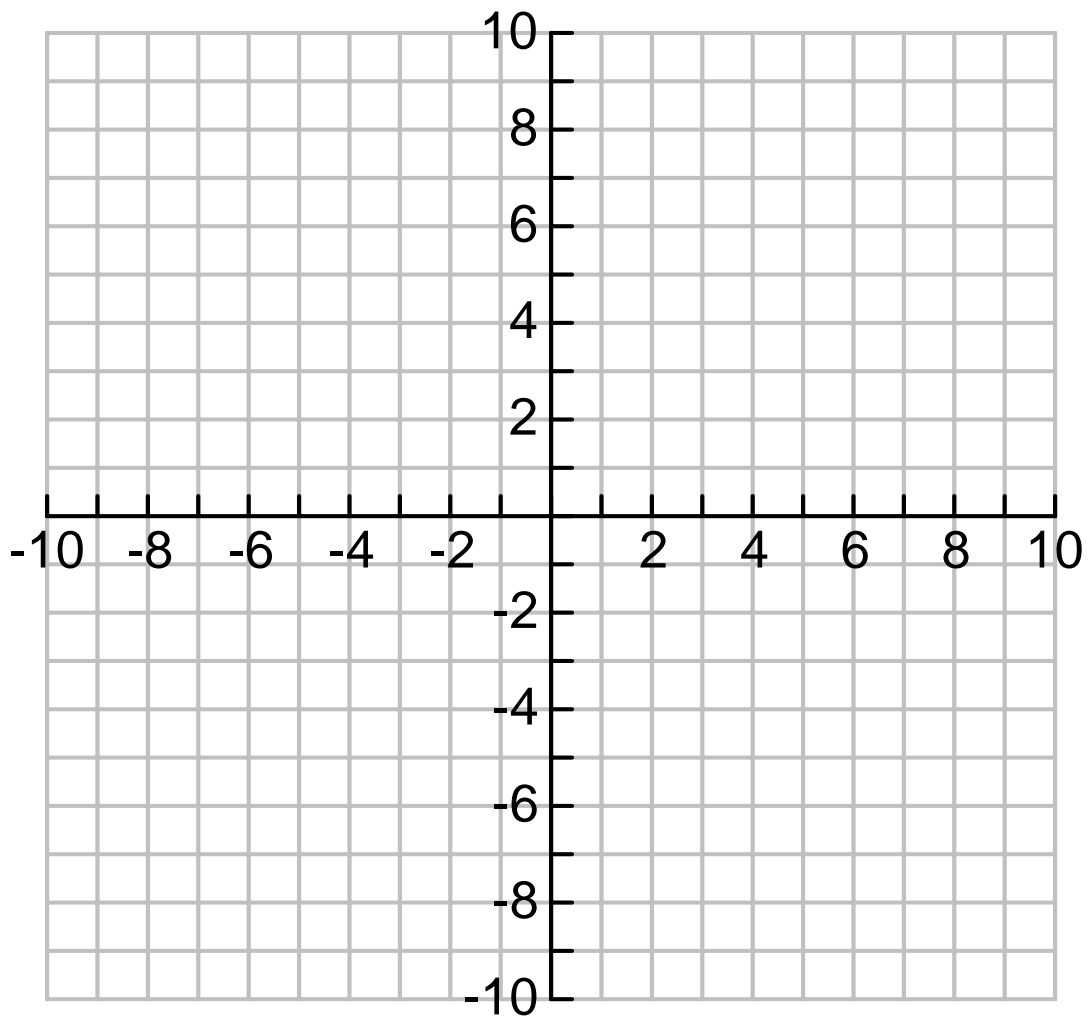
$4 + ? = 17$ $? = \underline{\quad}$	$34 - ? = 17$ $? = \underline{\quad}$	$? + 32 = 45$ $? = \underline{\quad}$	$? - 34 = 16$ $? = \underline{\quad}$
$34 + ? = 132$ $? = \underline{\quad}$	$83 - ? = 28$ $? = \underline{\quad}$	$? + 86 = 162$ $? = \underline{\quad}$	$? - 43 = 129$ $? = \underline{\quad}$
$? - 43 = 54$ $? = \underline{\quad}$	$54 + ? = 345$ $? = \underline{\quad}$	$? + 34 = 156$ $? = \underline{\quad}$	$83 - ? = 10$ $? = \underline{\quad}$
$123 - ? = 89$ $? = \underline{\quad}$	$390 + ? = 764$ $? = \underline{\quad}$	$? - 43 = 100$ $? = \underline{\quad}$	$? + 47 = 266$ $? = \underline{\quad}$
$739 - ? = 384$ $? = \underline{\quad}$	$? + 283 = 538$ $? = \underline{\quad}$	$978 + ? = 1,298$ $? = \underline{\quad}$	$? - 483 = 756$ $? = \underline{\quad}$

Open Sentence Practice Sheet 2

$3 \times ? = 24$ $? = \underline{\quad}$	$45 \div ? = 5$ $? = \underline{\quad}$	$? \times 2 = 8$ $? = \underline{\quad}$	$? \div 8 = 6$ $? = \underline{\quad}$
$? \times 7 = 84$ $? = \underline{\quad}$	$36 \div ? = 3$ $? = \underline{\quad}$	$? \div 2 = 13$ $? = \underline{\quad}$	$9 \times ? = 108$ $? = \underline{\quad}$
$? \div 5 = 20$ $? = \underline{\quad}$	$4 \times ? = 40$ $? = \underline{\quad}$	$? \times 8 = 184$ $? = \underline{\quad}$	$144 \div ? = 48$ $? = \underline{\quad}$
$16 \div ? = 4$ $? = \underline{\quad}$	$13 \times ? = 78$ $? = \underline{\quad}$	$? \times 8 = 168$ $? = \underline{\quad}$	$? \div 8 = 4$ $? = \underline{\quad}$
$? \times 18 = 72$ $? = \underline{\quad}$	$84 \div ? = 42$ $? = \underline{\quad}$	$8 \times ? = 96$ $? = \underline{\quad}$	$? \div 8 = 7$ $? = \underline{\quad}$

Rectangular Coordinate Grids

Four-quadrant Grid



Rectangular Coordinate Grids

One-quadrant grid

